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An Overview of ELECTRE Methods and their Recent Extensions

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ABSTRACT

We present main characteristics of ELECTRE (ELimination Et Choix Traduisant la REalité - ELimination and Choice Expressing the REality) family methods, designed for multiple criteria decision aiding. These methods use as a preference model an outranking relation on the set of actions—it is constructed in result of concordance and nondiscordance tests involving a specific input preference information. After a brief description of the constructivist conception in which the ELECTRE methods are inserted, we present the main features of these methods. We discuss such characteristic features as the possibility of taking into account positive and negative reasons in the modelling of preferences, without requiring commensurable performance scales; the use of discriminating thresholds for taking into account the imperfect knowledge of data; the absence of systematic compensation between 'gains' and 'losses'. The main weaknesses are also presented. Then, some aspects related to new developments are outlined. These are related to some new methodological developments, new procedures, axiomatic analysis, software tools and several other aspects. This paper is an updated version of a chapter published by the authors under the title 'ELECTRE Methods: Main Features and Recent Developments' in C. Zopounidis and P. Pardalos (Editors): *Handbook of Multicriteria Analysis*, Springer, Berlin 2010, pp. 51–89.

KEY WORDS: outranking methods; ELECTRE methods; survey

1. INTRODUCTION

Since their conception, which started in the 1960s of the last century, ELECTRE (ELimination Et Choix Traduisant la REalité - ELimination and Choice Expressing the REality) methods have been widely used for Multiple Criteria Decision Aiding (MCDA) in many real-world decision problems, ranging from agriculture to environment and water management, from finance to project selection, from personnel recruiting to transportation, and many others. The theoretical research on the foundations of ELECTRE methods has also been intensive all this time.

We believe that it is a right time to make comprehensive characteristics of ELECTRE methods with an emphasis on their recent extensions. The paper starts with philosophical considerations in Section 2, which underline the foundations of ELECTRE methods. Then, in Section 3, we describe characteristic features of ELECTRE methods, including modelling of preferences by outranking relations (Subsection 3.1), construction of the outranking relation using the concepts of concordance and discordance (Subsection 3.2), aggregation and exploitation procedures of ELECTRE methods (Subsection 3.3), strong features (Subsection 3.4) and weaknesses (Subsection 3.5) of ELECTRE methods. Section 4 is devoted to description of recent developments, including such issues as inferring model parameters and robustness issues (Subsection 4.1), improvements and new approaches (Subsection 4.2), axiomatic and meaningfulness analysis (Subsection 4.3), and other relevant issues (Subsection 4.4). Section 5 contains conclusions.

2. THE CONSTRUCTIVIST CONCEPTION OF MULTIPLE CRITERIA DECISION AIDING

In this next section, we explain what we mean by constructivist or 'European' conception of MCDA. More details on this issue can be found in Roy

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(2010). The term 'European' does not mean, however, that this conception was only developed and followed by Europeans. A large number of researchers all over the globe are being working in this area and are applying the techniques to real-world problems, for example, in Canada, Tunisia, Poland, Switzerland, Italy, Spain, Portugal, Germany, New Zealand and many other countries.

Before introducing the constructivist or 'European' conception of an MCDA methodology, we should present the meaning of a decision-aiding situation and its key elements, and three fundamental pillars that support such a conception. In what follows, the term 'decision aiding', rather than 'decision support', 'decision making' or 'decision analysis', will be adopted for escaping from simplistic assimilations. Thus, we claim that 'decision aiding' reflects better the constructive perspective of the decision process, whereas the other names are closer to normative and prescriptive perspectives (Bell *et al*., 1988).

2.1. Decision-aiding situation/decision-aiding process

Consider a company or a public institution, where a manager and/or a group of people, called stakeholders, are confronted with a decision situation or 'problem' that requires them to make a decision. They call on an in-house operational research service or an outside consultant or even a university research team to obtain help in the decision process. More precisely, a help in clarifying the decision and recommending, or simply favouring, a behaviour that will increase the consistency between the evolution of the process and the stakeholders' objectives and value systems. This aspect allows to characterize a *decision-aiding situation*, where two key actors are relevant for co-interaction that will lead to build and make evolve the *decision-aiding process*; a process that comprises several phases (Roy, 1996). The two key actors will be designated in what follows as the *analyst*, who is appointed to give this decision aiding, and as the *decision maker*, in whose name or for whom this decision aiding is to be given.

2.2. Three fundamental pillars

In the operational research and decision-aiding community, to which we belong, the *decision-aiding activity* (which is meant to be scientific) is founded on three pillars:

1. The *actions* (the formal definition of the objects of the decision, called alternatives when two distinct actions cannot be put conjointly into operation),
2. The *consequences* (aspects, attributes, characteristics, ... of the actions, that allow to compare one action to another), and
3. The *modelling of one or several preference systems* (it consist of an implicit or explicit process, that for each pair of actions envisioned, assigns one and only one of the three situations (see Subsection 3.1): *indifference*, *preference* or *incomparability*).

The last pillar needs further explanation. When given two possible actions, any individual, whoever he or she may be, on the basis of the actions' consequences, and his and her value system, can state: 'I prefer the first to the second' or *vice versa*, 'I am indifferent between the two', or 'I am unable to compare these two actions'.

Modelling a preference system means to specify a process that will provide this type of results on the basis of a pre-established model of the action consequences. These consequences are most often complex and imperfectly known. They can be modelled in quantitative or qualitative terms, in a deterministic or stochastic manner, with a part of arbitrariness or ill determination. We will designate by $C(a)$ the model of the consequences of action a.

2.3. The 'European' conception of Multiple Criteria Decision Aiding

According to the 'European' conception, the analyst must seek for obtaining *a coherent and structured set of results*. These results should be sought to guide the decision-aiding process and facilitate communication about the decision. To do so, the analyst must use an approach that aims at *producing knowledge from working hypotheses*, taking into account the objectives and the value systems involved in a particular decision context. This approach should be based on models that are, at least partially, *co-constructed through interaction* between the analyst and the decision maker (the prefix 'co' stems for the interaction between decision maker and analyst). This co-construction first concerns the way the considered actions are taken into account, as well as the actions consequences on which they will be judged. Secondly, the *co-construction process* concerns the way that certain characteristics (notably the values attributed to the different parameters) of the preference model were judged the most appropriate given the specificities of the decision context and the working hypotheses retained. In this conception, it is not necessary to assume that there exists, in the mind of the decision maker, a stable procedure capable of defining the decision maker's

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preference system, before even beginning the decision-aiding process as in Multiple Attribute Utility Theory methods (Keeney & Raiffa, 1976).

To elaborate results likely to make things more clear to the decision maker (e.g. 'if…, then…' results), in the 'European' conception, the analyst must propose working hypotheses that will allow the co-construction of the preference model to play an appropriate role in the decision-aiding process. The co-constructed model must be a tool for looking more thoroughly into the subject, by exploring, interpreting, debating and even arguing. To guide this process of co-construction, the analyst must interact with the decision maker assuming that he or she understands the questions that are asked. Nevertheless, in the 'European' conception, it is not necessary to assume that the given responses are produced through a stable pre-existing process, but only that these responses are made up through interaction with the decision maker's value system, which is *rarely free of ambiguity or even contradiction*. In particular, the analyst must make sure that the person who responds to the questions is able to place these questions in the context of the current study. The analyst must also admit that these questions can bring the person thus questioned to revise certain pre-existing preferences momentarily and locally.

According to the 'European' conception, the knowledge produced does not aim to help the decision maker to discover a good approximation of a decision that would objectively be one of the best, taking into account his or her own value system, but rather more humbly to provide the decision maker with a set of recommendations derived from the reasoning modes and working hypotheses. The decision maker will better understand the recommendations produced and will appropriate them (and potentially share with others) if the analyst makes sure that understanding of the underlying reasoning modes and working hypotheses is integrated into the model co-construction process.

In this 'European' conception, the analyst does not need to accept either of the following two postulates that are often implicitly admitted in other conceptions (Roy, 2010):

- *Postulate of the decision maker's optimum*. In the decision context studied, there exists at least one optimal decision, or, in other words, there exists one decision for which it is possible (if sufficient time and means are available) to establish objectively that there are no strictly better decisions with respect to the decision maker's preference system.
- *Postulate of the decision context reality*. The principal aspects of the reality on which the decision aiding is based (particularly the decision maker's preferences) are related to objects of knowledge that can be seen as data (i.e. existing outside of the way they are modelled); these objects can also be seen as sufficiently stable over time and for the questions asked, such that it is possible to refer to the exact state or the exact value (deterministic or stochastic) of given characteristics judged to accurately portray an aspect of that reality.

He or she may find these postulates as totally unrealistic, or may even have good reasons for accepting the existence of *incomparabilities* in the preference models used.

Concluding, in this constructivist conception, the source of legitimization is situated in procedural rationality and communication, and not in the hypothesis of a rational decision maker who is reasoning in conformity with a set of axioms. The approach to decision aiding is based on one or more preference models co-constructed with the decision maker to study the results to which they lead. This approach is not looking for faithfully reproducing the decision maker's preference system, that is supposed to pre-exist, to obtain as close as possible to the best decision. We finally admit that once the analyst enters into interaction with the decision maker, this interaction makes him or her a co-constructor of the knowledge produced; thus, (s)he cannot be seen as being outside of the decision-aiding process.

3. MAIN FEATURES

The distinctive features of ELECTRE methods, to which analysts should pay special attention on, when dealing with real-world decision-aiding situations, are presented in this section. These are the four preference situations handled by ELECTRE methods, the preference modelling by outranking relations, the concepts of concordance and discordance, the scheme of the decision-aiding method, the main strengths as well as the weaknesses of ELECTRE methods.

For a suitable description of the main features and recent developments of ELECTRE methods, it is necessary to introduce a few notation related to the basic data.

It should be noticed that numbers used to code preferences have an ordinal meaning only. Consequently, the difference between the performances of two actions on any criterion must not be considered

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as an intensity of preference (for more details see Section 3.2.1).

The basic data needed for any MCDA problem can be represented as follows:

1. $A = \{a_1, a_2, \ldots, a_i, \ldots, a_m\}$ is the set of *m potential actions*; this set is, possibly, only partially known *a priori*, which is common in sorting problems (see Section 3.2),
2. $F = \{g_1, g_2, \ldots, g_j, \ldots, g_n\}$ is a *coherent family of criteria* with $n \geqslant 3$ (for the properties of a coherent family of criteria see Roy (1996)),
3. $g_j(a_i)$ is the *performance* of action a_i on criterion g_j, for all $a_i \in A$ and $g_j \in F$; an $m \times n$ *performance matrix M* can thus be built, with $g_j(a_i)$ in row i and column $j (i = 1, \ldots, m; j = 1, \ldots, n)$.

Let w_j denote the relative importance coefficient of criterion g_j, for all $g_j \in F$ (assume, without loss of generality $\sum_{j=1}^{n} w_j = 1$). This coefficient can be viewed as an intrinsic weight: it can be interpreted as the voting power of each criterion. The higher the intrinsic weight, the more important the criterion is. Note that the voting power neither depends on the range of the criterion scale nor on the encoding chosen (in particular the unit selected) to express the performance of an action on this scale.

In the following, we assume without loss of generality that the higher the performance $g_j(a)$ is, the better it is for the decision makers (*increasing direction of preference*).

3.1. Preference modelling by outranking relations

This subsection is devoted to preference model, the introduction of the pseudo-criterion model, and the binary outranking relation.

3.1.1. Modelling of four main preference situations. The ELECTRE methods are handling the following four preference situations concerning the comparison of two actions (Roy, 1996):

- (*Indifference*): it corresponds to a situation where there are clear and positive reasons that justify an equivalence between the two actions (it leads to a reflexive and symmetric but not necessarily transitive binary relation);
- (*Strict Preference*): it corresponds to a situation where there are clear and positive reasons in favour of one (identified) of the two actions (it leads to a nonreflexive and asymmetric and usually transitive binary relation);
- (*Weak Preference*): it corresponds to a situation where there are clear and positive reasons that invalidate strict preference in favour of one (identified) of the two actions, but they are insufficient to deduce either the strict preference in favour of the other action or indifference between both actions, thereby not allowing either of the two preceding situations to be distinguished as appropriate (it leads to a nonreflexive and asymmetric but not usually transitive binary relation);
- (*Incomparability*): it corresponds to an absence of clear and positive reasons that would justify any of the three preceding relations (it leads to a nonreflexive and symmetric binary relation).

Notice that the meaning of 'clear and positive reasons' is related to the concepts of concordance and nondiscordance defined in Subsection 3.2.

3.1.2. The concept of pseudo-criterion.

Definition 1

(*pseudo-criterion*) A pseudo-criterion is a real-valued function g_j associated with two threshold functions, $q_j(\cdot)$ and $p_j(\cdot)$, satisfying the following condition: for all ordered pairs of actions $(a, a') \in A \times A$, such that $g_j(a) \geqslant g_j(a')$, $g_j(a) + p_j(g_j(a'))$ and $g_j(a) + q_j(g_j(a'))$ are nondecreasing monotone functions of $g_j(a')$, such that $p_j(g_j(a')) \geqslant q_j(g_j(a')) \geqslant 0$ for all $a \in A$.

For more details about the concept of pseudo-criterion see Roy (1991) and Roy and Vincke (1984). Here, we consider the thresholds as variables, but they can also be defined as constant values. Moreover, not necessarily all the criteria are subject to the definition of indifference and preference discriminating thresholds, which are used to distinguish between indifference, weak preference and strict preference situations when comparing two actions (see Subsection 3.1.3). It should also be noted, that the way a pseudo-criterion was defined previously, takes into account only *direct thresholds*, because the arguments of the threshold functions are the worst of the two performances $g_j(a)$ and $g_j(a')$. When the thresholds are expressed as a function of more preferred of the two values, we call them *inverse thresholds*. In the case of constant thresholds, there is no distinction between direct and inverse thresholds.

According to the aforementioned definition,

- $q_j(g_j(a'))$ is the greatest performance difference for which the situation of indifference holds on criterion g_j between two actions a and a', where $q_j(g_j(a')) = g_j(a) - g_j(a')$,

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- $p_j(g_j(a'))$ is the smallest performance difference for which the situation of preference occurs on criterion g_j between two actions a and a', where $p_j(g_j(a')) = g_j(a)$ $g_j(a')$.

The reader can find more details about the discriminating thresholds in Roy (1985, 1996).

3.1.3. The definition of the partial binary relations. Consider an ordered pair of actions $(a,a') \in A \times A$, and the two thresholds associated with the pseudo-criterion $g_j \in F$, which is used to model the following situations (note that no assumption is made here about which one of the two actions is better on criterion g_j):

(1) $g_j(a) - g_j(a') > p_j(g_j(a'))$	$\Leftrightarrow$	aP_ja',
(2) $q_j(g_j(a')) < g_j(a) - g_j(a') \leqslant p_j(g_j(a'))$	$\Leftrightarrow$	aQ_ja' (hesitation between aI_ja' and aP_ja'),
(3) $-q_j(g_j(a)) \leqslant g_j(a) - g_j(a') \leqslant q_j(g_j(a'))$	$\Leftrightarrow$	aI_ja'.

The aforementioned three binary relations can be grouped into one partial outranking relation S_j comprising the three corresponding situations $S_j = P_j \cup Q_j \cup I_j$, where aS_ja' means that 'a is at least as good as a'' on criterion g_j. When aS_ja', the whole voting power, w_j, of criterion g_j is considered. Figure 1 illustrates the different zones of the partial outranking relations previously defined, that is, the situations $a'P_ja$, $a'Q_ja$, aI_ja', aQ_ja' and aP_ja' as well as the fraction φ_j of the voting power associated with each one of these situations.

From the definition of the partial binary relations and from Figure 1, it is easy to see that the two types of thresholds, direct and inverse, have to be taken into account.

3.1.4. The comprehensive outranking relation. Preferences in ELECTRE methods are modelled by the comprehensive binary outranking relation S, whose meaning is '*at least as good as*'; in general, $S = P \cup Q \cup I$.

Consider two actions $(a,a') \in A \times A$. Modelling comprehensive preference information leads to the four cases ($\succ = Q \cup P$):

1. aSa' and not$(a'Sa)$, that is, $a \succ a'$ (a is *weakly or strictly preferred* to a');
2. $a'Sa$ and not(aSa'), that is, $a' \succ a$ (a' is *weakly or strictly preferred* to a);
3. aSa' and $a'Sa$, that is, aIa' (a is *indifferent* to a');
4. not(aSa') and not$(a'Sa)$, that is, aRa' (a is *incomparable* to a').

It is worth stressing that, taking into account the properties of its component relations, the outranking relation is not transitive in general.

3.2. The concepts of concordance and discordance

All outranking based methods rely on the concepts of concordance and discordance that represent, in a certain sense, the raisons *for* and *against* an outranking situation.

3.2.1. Concordance. Concordance refers to the strength of the coalition of criteria being in favour of the outranking relation aSa'.

3.2.1.1. The comprehensive concordance index. The *comprehensive concordance index* can be defined as follows:

$$c\left(a, a'\right) = \sum_{j \in \mathcal{C}^S\left(a,a'\right)} w_j + \sum_{j \in \mathcal{C}^Q\left(a',a\right)} w_j \varphi_j,$$

where

$$\varphi_j = \frac{g_j(a) - g_j\left(a'\right) + p_j\left(g_j(a)\right)}{p_j\left(g_j(a)\right) - q_j\left(g_j(a)\right)},$$

and $\mathcal{C}^S\left(a, a'\right) = \left\{j : aS_ja'\right\}$ are the coalition of criteria that are in favour of the assertion aSa' with

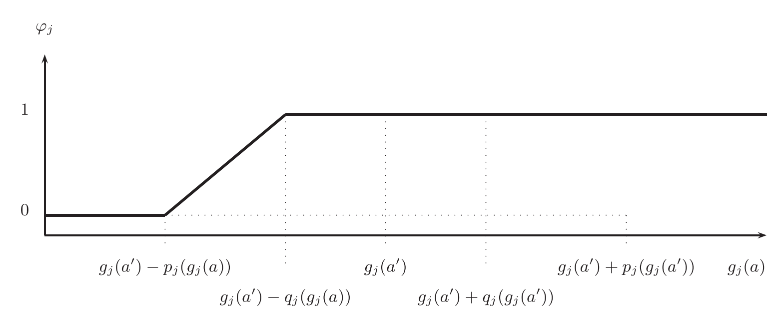

Figure 1. Variation of φ_j for a given $g_j(a')$ and variable $g_j(a)$.

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no reservation, and $\mathcal{C}^{Q}\left(a,a'\right)=\left\{j:a'Qa\right\}$ is the coalition of criteria that hesitate between the indifference and the opposition.

The $m\times m$ concordance matrix C is composed of elements $c(a,a')$, for all $a,a'\in A$.

One can see that the criteria can be classified in three groups:

1. those that are in favour of the assertion aSa' with no reservation,
2. those that hesitate between the indifference and the opposition,
3. those that are in the opposition.

The $c(a,a')$ index results from the summation of the voting power of criteria from the first group, and of the fraction φ_j of the voting power of criteria from the second group (see also Section 1). This fraction can be interpreted as the proportion of the voting power (the weight of criterion g_j) in favour of the assertion aSa'. Such a proportion is close to 1 when the hesitation is 'closer' to indifference, and it is close to 0, when the hesitation is 'closer' to strict preference. The function φ_j is a monotone nonincreasing function of the difference between $g_j(a)$ and$g_j(a')$ in the interval between indifference and preference thresholds. In case of a quantitative criterion, this difference is calculated in a straightforward way. In case of a qualitative criterion, each scale level of an ordinal scale is to be number coded in a way respecting the scale level, as well as taking correctly into account the indifference and preference thresholds; then, the difference has also a numerical value. When the difference in favour of a' is not greater than the indifference threshold, the value of $\varphi_j(a,a')$ is necessarily equal to 1, and when this difference is at least as large as the preference threshold, the value of $\varphi_j(a,a')$ is necessarily equal to 0. The interval between the indifference and the preference thresholds corresponds to an ambiguity zone in which one hesitates between indifference aI_ja' and strict preference $a'P_ja$. It should be small comparing with the spread of the criterion scale. Even if this could be perceived as somehow arbitrary, it is enough to adopt the simplest linear form for this monotone function φ_j. Remark that this convention has nothing in common with a statement of intensity of preference. Adopting a slightly concave or convex form for φ_j would rather increase the part of arbitrary. Anyway, our experience indicates that slight changes of the form of this monotone function have no impact (apart from very particular cases) on the results.

From the viewpoint of the concordance, we can say that aSa', whenever $c(a,a')\geqslant s$, where s is a concordance threshold.

3.2.2. Discordance. Discordance refers to criteria that are in opposition to the assertion aSa'.

3.2.2.1. The concept of veto threshold. When criterion g_j opposes strongly to the assertion aSa', g_j puts its veto to this assertion. This occurs if $g_j(a')-g_j(a)>v_j(g_j(a))$. The value $v_j(g_j(a))>p(g_j(a))\geqslant 0$ is called the veto threshold of g_j.

Veto effect can be weakened to avoid a rigid binary situation in which a criterion imposes a veto or not. ELECTRE methods can handle such situations through the partial discordance indices of criteria.

3.2.2.2. Partial discordance indices.

$$d_j\left(a,a'\right)=\begin{cases}1 & \text{if } g_j(a)-g_j\left(a'\right)<-v_j\left(g_j(a)\right),\\ \dfrac{g_j(a)-g_j\left(a'\right)+p_j\left(g_j(a)\right)}{p_j\left(g_j(a)\right)-v_j\left(g_j(a)\right)} & \text{if } -v_j\left(g_j(a)\right)\leqslant g_j(a)-g_j\left(a'\right)<-p_j\left(g_j(a)\right),\\ 0 & \text{if } g_j(a)-g_j\left(a'\right)\geqslant -p_j\left(g_j(a)\right).\end{cases}$$

where $d_j(a,a')$ is the partial discordance index of criterion g_j.

It permits to build an $m\times m$ discordance matrix D_j composed of elements $d_j(a,a')$, for all $(a,a')\in A\times A$ and for each criterion $g_j\in F$.

It is worth noting that in ELECTRE methods, it is not assumed that the weights, as well as the veto thresholds, have a real existence in the mind of the decision maker. They do not have a 'true value'.

Such parameters are artefacts, co-constructed abstract 'objects' (Roy, 2010).

3.3. Aggregation and exploitation procedures of ELECTRE methods

Each ELECTRE method comprises two main procedures: an aggregation procedure and an exploitation procedure.

3.3.1. Multiple Criteria Aggregation Procedures. By definition, MCAP is a procedure that builds one or possibly several outranking relations on the basis of the performances of each action on each criterion, which leads to assign to each ordered pair of actions one and only one of the four situations presented in Subsection 3.1. Let us notice that the decision maker does not make any pairwise comparison; all the comparisons are carried out by the procedure itself.

The MCAP has to take into account the role played by the criteria: some of them can play a 'very important' role, whereas others can play a 'totally secondary' role. For this purpose, ELECTRE methods make use of intrinsic weights and possible veto thresholds (Figueira *et al*., 2005, chap. 4).

For example, in ELECTRE III, its MCAP associates to each ordered pair of actions $(a,a') \in A \times A$ a credibility index of the assertion aSa'. This credibility index, denoted by $\sigma(a,a') \in [0,1]$, can be interpreted as a degree of credibility, which synthesizes the strength of the coalition of criteria being in favour of the assertion aSa' with the opposition of criteria being against this assertion. Thus, it combines $c(a,a')$ and $d_j(a,a')$ in the following way:

$$\sigma\left(a,a'\right) = c\left(a,a'\right)\prod_{j=1}^{n} T_j\left(a,a'\right),$$

where $T_j\left(a,a'\right) = \frac{1-d_j\left(a,a'\right)}{1-c\left(a,a'\right)}$ if and only if $d_j(a,a') > c(a,a')$, and $T_j(a,a') = 1$ otherwise.

The rationale underlying this index can be found in Roy and Bouyssou (1993).

It should be noticed, moreover, that other possible ways of defining this index have been proposed for instance by Mousseau and Dias (2006).

The fuzzy relation $\sigma(a,a')$ is frequently converted into a crisp relation through the use of a λ-cutting level.

Let us observe that aggregation procedures used in ELECTRE methods are recommended to be used in decision-aiding situations involving more than four criteria.

3.3.2. Exploitation procedures and the type of the results. ELECTRE methods involve different exploitation procedures. Each exploitation procedure (EP) is adapted to a particular problematique (see Roy, 1996, chap. 6). In MCDA, a problematique concerns the way in which decision aiding is conceived, and, more precisely, the type of results that must be provided by the method to carry out decision aiding. The three major problematiques in MCDA can be stated as follows:

3.3.2.1. Choosing. Selecting a restricted number of the most interesting potential actions, as small as possible, which will justify to eliminate all others.

3.3.2.2. Sorting. Assigning each potential action to one of the categories among a family previously defined; the categories are ordered, in general, from the worst to the best one. An example of a family of categories suitable for assignment procedures is given as follows:

C_1: actions whose implementation is not advised;
C_2: actions whose implementation could only be advised after significant modifications;
C_3: actions whose implementation could only be advised after slight modifications;
C_4: actions whose implementation is always advised without any reservation.

3.3.2.3. Ranking. Ranking of actions from the best to the worst, with the possibility of ties (*ex æquo*) and incomparabilities.

3.3.2.4. Remark.

1. In *sorting problematique*, the result depends on *absolute evaluation* of actions: the assignment of an action takes into account, *only its intrinsic evaluation on all the criteria*, and it *neither depends on nor influences* the category to be selected for the assignment of another action.
2. As in the *remaining problematiques*, the actions are compared against each other; the result depends in these cases on *relative evaluation* instead of absolute one as in the previous case.

Observe that the results provided by the method will then be used by an analyst to work out his or her recommendations.

3.3.3. Main ELECTRE methods. The family of ELECTRE methods includes several methods designed for the three main problematiques defined previously (see Figueira *et al*., 2005, Subsection 3.2):

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1. Choosing: ELECTRE I, ELECTRE Iv (read 'ELECTRE one vee') and ELECTRE IS (read 'ELECTRE one S'). ELECTRE IS is a generalization of ELECTRE Iv.
2. Ranking: ELECTRE II (read 'ELECTRE two'), ELECTRE III (read 'ELECTRE three') and ELECTRE IV (read 'ELECTRE four').
3. Sorting: ELECTRE TRI-B (originally called just ELECTRE TRI, read 'ELECTRE tree'), ELECTRE TRI-C and ELECTRE TRI-NC.

3.4. Strong features

This subsection goes through major strong features of ELECTRE family methods. They include the possibility of dealing with the qualitative as well as the quantitative scales of criteria. The heterogeneity of scales and the nonrelevance of compensatory effects are also discussed here. The imperfect knowledge of data and some arbitrariness when building criteria can be taken into account in ELECTRE methods, and, finally, they can deal with the reasons for and against an outranking.

3.4.1. The qualitative scales of some criteria. ELECTRE family methods are able to handle qualitative performance scales of criteria. They allow to consider original (verbal or numeric) performances, without the need of any recoding. In fact, all the criteria are processed as qualitative criteria, even if some are quantitative by their very nature.

3.4.2. The heterogeneity of scales. The ELECTRE family methods can deal with *heterogeneous* scales to model such diversified notions as noise, delay, aesthetics, cost, ... Whatever the nature of scales, every procedure can run with preserved original performances of the actions on the criteria, without the need of recoding them, for example, by using a normalization technique or the assessment of the corresponding evaluations through the use of a utility or a value function.

3.4.3. The nonrelevance of compensatory effects. The MCAPs of ELECTRE methods were conceived such that they do not allow for compensation of performances among criteria, that is, the degradation of performances on certain criteria cannot be compensated by improvements of performances on other criteria.

The weights of criteria do not mean substitution rates as it is the case in many other methods. The limited possibility of compensation can be brought into light through the concordance and discordance indices:

- Concerning the concordance index, when comparing action a to action a', with the exclusion of the ambiguity zone, only the fact that a outranks or does not outrank a' with respect to criteria from F is relevant, whereas it is not relevant how much the performance of a is better or worse than the performance of a' on criteria from F;
- The existence of veto thresholds strengthening the noncompensatoriness effect is yet another reason of the possibility of noncompensation in ELECTRE methods. For example, when $d_j(a,a')=1$, no improvement of the performance of a and no deterioration of the performance of a', with respect to the other criteria than g_j, can compensate this veto effect.

Consider the following example with four criteria and only two actions (scales: [0,10]). The performance matrix for this example is given in Table 1. Suppose that the weighted-sum model was chosen, that is, $V(a)=w_1g_1(a)+\ldots+w_jg_j(a)+\ldots+w_ng_n(a)$. In the considered example, the weights w_j are equal for all criteria ($w_j=0.250$, for all $j=1,\ldots,4$):

$V(a_1)=8.125>V(a_2)=8.100$ (notice that $V(a_1)-V(a_2)=0.025$), and so a_1Pa_2 (a_1 is strictly preferred to a_2). The difference between the performances of the two actions is small on the first three criteria, whereas this difference on the fourth criterion (3.100) is very big in favour of a_2. The compensatory effect led a_1 to be strictly preferred to a_2. This example shows, in an obvious way, the possibility that a big preference difference not favourable to a_1 on one of the criteria (g_4) can be compensated by three differences of small amplitude on the remaining criteria, in such a way that a_1 becomes finally strictly preferred to a_2.

In ELECTRE methods, the type of compensatory effect shown in the aforementioned example does not occur in a systematic way (see Sections 4 and 5). Thus, contrarily to many other methods, (e.g. Choquet and Sugeno integrals), there is no need in ELECTRE methods to use identical and commensurable performance scales.

3.4.4. Taking into account the imperfect knowledge of data and some arbitrariness when building criteria. ELECTRE methods are adequate to take into account the *imperfect knowledge* of the data and the

Table I. Performance matrix

	g_1	g_2	g_3	g_4
a_1	9.500	9.500	8.100	5.400
a_2	8.300	8.300	7.300	8.500

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arbitrariness related to the construction of the family of criteria. This is modelled through the indifference and preference thresholds (discriminating thresholds). Consider the same example with the (constant) discriminating thresholds, given in Table 2.

On the one hand, it should be noticed that any small variation of some performance will not affect in a significant way the preference difference resulting from the MCAP used in ELECTRE methods, but it will modify the weighted-sum value. For example, if on criterion g_3, we would change the performance of action a_2 from 7.300 to 7.100, then the weighted-sum score $V(a_2)$ would move from 8.100 to 8.050 ($V(a_1) - V(a_2) = 0.075$). Consequently, there would be a reinforcement of the preference in favour of a_1.

On the other hand, in ELECTRE methods, $c(a_1,a_2)$ and $c(a_2,a_1)$ remain unchanged, as it will be shown hereafter. Because the weighted-sum based models do not allow for the inclusion of thresholds, a_1 is still better than a_2. Now, if we consider 7.500 instead of 7.300, then $V(a_2) = 8.150$, and consequently a_2Pa_1. This slight variation is really too small to invert the preference between a_1 and a_2, but because the weighted-sum based models do not allow for the inclusion of thresholds, a_2 became preferred to a_1. This phenomenon shows the sensitivity of the weighted-sum with respect to non-significant variations of the performances, because of the compensatory character of the model.

The performances of the actions can be affected by the imperfect knowledge coming from different sources. At the same time, the way the criteria are built or conceived contains some part of arbitrariness. These are the two major reasons that led to define the discriminating thresholds in ELECTRE methods. When considering the discriminating thresholds and using ELECTRE methods, $c(a_1,a_2) = 0.250 + 0.250 + 0.250 = 0.750$ and $c(a_2,a_1) = 0.200 + 0.200 + 0.250 + 0.250 = 0.900$ (see Section 3.2.1).

3.4.5. Reasons for and reasons *against* an outranking. The ELECTRE methods are based, in a certain sense, on the *reasons for* (concordance) and the *reasons against* (discordance) of an outranking between two actions. Consider the same example and a veto threshold $v_j = 3$, for all $j = 1, \ldots, 4$ (see Table 3).

Table II. Performances and discriminating thresholds

	g_1	g_2	g_3	g_4
a_1	9.500	9.500	8.100	5.400
a_2	8.300	8.300	7.300	8.500
q_j	1.000	1.000	1.000	1.000
p_j	2.000	2.000	2.000	2.000

If the concordance threshold $s = 0.800$, then a_2Sa_1 and *not*(a_1Sa_2). But, if $s = 0.700$, then a_2Sa_1 and a_1Sa_2, that is, a_2Ia_1.

The discordance index (see Section 3.2.2) of g_4, $d_4(a_1,a_2) = 1$, and whatever the value of concordance threshold s, we obtain *not*(a_1Sa_2). This means that g_4 imposes its veto power on the assertion a_1Sa_2. Weighted-sum based models do not allow for the inclusion of veto effects.

The aforementioned shows, moreover, that the consideration of a veto threshold reinforces the non-compensatory character of the ELECTRE methods.

3.5. Weaknesses

This subsection gives account of the main drawbacks or weaknesses of ELECTRE methods, notably when the quantitative nature of the family of criteria requires the use of a different method, when a score should be assigned to each action, when the independence with respect to irrelevant actions and the possible instability of the set of actions is required, or the possible and frequent occurrence of intransitivities would make a problem.

3.5.1. Scoring of actions. In certain contexts, it is required to assign a *score* to each action. When the decision makers require that each action should obtain a score, the ELECTRE methods are not adequate for such a purpose, and the scoring methods should be applied instead. The decision makers should be aware, however, that using a score method, they cannot provide information that leads, for example, to intransitivities or to incomparabilities between some pairs of actions. Moreover, the score is very fragile.

For the time being, there is no outranking-like method allowing to assign a score to different actions in a convincing manner. This seems to be a very difficult issue because it is assumed to measure preference difference (or intensity of preference). In PROMETHEE (Preference Ranking Organization METHod for Enrichment Evaluations) (Brans & Mareschal, 2005, chap. 5), there

Table III. Performances, discriminating and veto thresholds

	g_1	g_2	g_3	g_4
a_1	9.500	9.500	8.100	5.400
a_2	8.300	8.300	7.300	8.500
q_j	1.000	1.000	1.000	1.000
p_j	2.000	2.000	2.000	2.000
v_j	3.000	3.000	3.000	3.000

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was an attempt to define a measure of preference differences, but the way in which it was presented seems to contain matter for some criticism (see Section 6.4.1 in Roy & Bouyssou, 1993).

3.5.2. The quantitative nature of the family of criteria. When all the *criteria are quantitative*, it is 'better' to use some other methods. But, if we would like to take into account a completely or even a partially noncompensatory method, as well as the reasons for and against, then, even if the criteria would be all quantitative, we should use ELECTRE methods. Assume that all the criteria are quantitative and defined on the same scale with the same unit. Also, then, if we are dealing with imperfect knowledge with respect to at least one criterion, ELECTRE methods are suitable.

3.5.3. The independence with respect to irrelevant actions. Except ELECTRE TRI-B, ELECTRE TRI-C and ELECTRE TRI-NC, the remaining ELECTRE methods do not fulfil the property of *independence with respect to irrelevant actions*, which says that when comparing two actions, the preference relation should not depend on the presence or absence of other actions. Roy (1973) shows that rank reversal may occur and, consequently, the property of independence with regard to irrelevant actions can be violated when dealing with outranking relations. Notice that rank reversal may occur only when the set of potential actions is subject to evolve, which is quite a natural assumption, however, it is not present in many real-world decision-aiding processes where the number of actions is rather small and easily identified. Roy (1973) presents an example illustrating that such phenomena can be interpreted quite naturally, and the author also suggests that allowing the independence property is not realistic in many real-world decision-aiding situations. Other works devoted to the same kind of concern include, for example, Perny (1992), Roy and Bouyssou (1993), Simpson (1996) and Wang and Triantaphyllou (2008).

In fact, the *instability* of the results in ELECTRE methods was recently re-analysed by Wang and Triantaphyllou (2008) with respect to ELECTRE II and III. When the decision makers feel more comfortable and confident with an evaluation model that provides a stable result, they might be a little bit surprised by the results provided by ELECTRE methods in certain circumstances. In our perspective, a stable result is not necessarily the evidence of an adequate processing of data because some aggregation procedures assume that the data have a meaning, but very often, they do not really have it. For example, this is often the case of the weighted-sum based methods, where the results may be stable but not necessarily meaningful (Martel & Roy, 2006). Moreover, if one uses different normalization procedures (as is the case when one deals with multiple units of measurement), such methods may alter the derived results (Triantaphyllou, 2000). What the ELECTRE methods show is related to the poorly determined margins on the results, very often related to the poor quality of data because the scales are processed as ordinal ones.

Regarding the rank reversal it is important to underline the following aspects:

1. It is quite natural that MCAPs based on pairwise comparisons violate the principle of independence with respect to irrelevant actions. The possibility of what is called rank reversal is a consequence of this violation.
2. In ELECTRE methods, when there exists a phenomenon of rank reversal between action a and action a', this sheds some light on the fact that the way a and a' are compared is not robust. This is due to the following two reasons:
   - the existence of discriminating thresholds and the values that should have been assigned to them,
   - the fact that such a comparison is conditioned by the way the actions a and a' are compared with the remaining actions (Wang & Triantaphyllou, 2008; Figueira & Roy, 2009).

3.5.4. Intransitivities. *Intransitivities* may also occur in ELECTRE methods (Roy, 1973). It is also well known that methods using outranking relations (not only the ELECTRE methods) do not need to satisfy the transitivity property. This aspect represents a weakness only if we impose *a priori* that preferences should be transitive. There are, however, some reasons for which the transitivity should not be imposed:

1. It is quite natural that the binary relation of indifference should be considered intransitive (see an example illustrating this phenomenon in Luce (1956)); there is also no reason to avoid defining indifference thresholds for certain criteria.
2. It is also possible to have insensitivities with respect to the binary relation of preference; we would say that it is possible and rather frequent to have a majority of the criteria in favour of a over b, and majority of the criteria of b over c, without necessarily implying that there is a majority of the criteria in favour of a over c; we can also have a majority of criteria in favour of c over a; this is

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the well-known Condorcet Paradox, described, for example, in Bouyssou *et al*. (2000). In fact, Gerhlein (1983) proved that for 25 voters and 11 candidates, the probability that the Condorcet Paradox occurs is 50%.

Let us notice that there is no such intransitivity phenomenon in ELECTRE TRI-B and ELECTRE TRI-C methods.

3.5.5. Discussion. A detailed discussion of the weak and the strong points of ELECTRE methods can be found in Figueira and Roy (2009). The objective of this discussion was to draw the attention of the readers to a particular philosophy that should be used to interpret correctly the results of ELECTRE methods. It is different from philosophies typically applied to interpret results of other methods. Ignoring this difference may lead to a misunderstanding (Wang & Triantaphyllou, 2008). In Figueira and Roy (2009), as well as in Roy (1996), the authors try to show, that the objective of decision aiding is not to discover an absolute truth or, a pre-existing 'real' best action, ranking or assignment. The modifications that may occur when adding or removing an action emphasizes the limitations of the conclusions that can be derived by using ELECTRE methods when one is using their results without an appropriate robustness analysis. Clearly, this is also what decision aiding is designed to do: to show how the conclusions can be drawn without claiming to reveal a pre-existing truth.

4. RECENT DEVELOPMENTS

The recent developments presented in this section are mainly methodological. They concern new approaches, an axiomatic analysis of ELECTRE methods, as well as some aspects related to the meaningfulness of the methods.

4.1. Inferring model parameters and robustness issues

This subsection is devoted to the presentation of the inference-based approaches and some related issues, the inference-robustness based approaches, the pseudo-robustness based approaches and the new concepts of robustness that can be applied to ELECTRE methods.

4.1.1. Pure inference-based approaches. Mousseau and Słowiński (1998) propose the first general algorithm for inferring the values of the model parameters of ELECTRE TRI-B method from assignment examples given by the decision maker, that is, from holistic judgments. Assignment examples serve to build a set of mathematical constraints, and the inference of the model parameters consists in solving a mathematical programming problem. This approach represents the paradigm of disaggregation–aggregation of preferences (Jacquet-Lagrèze & Siskos, 1982), which aims at extracting implicit information contained in holistic statements given by a decision maker. In this case, the statements to be disaggregated are assignment examples. Such an indirect elicitation of preferences requires from the decision maker a much smaller cognitive effort than direct elicitation of the model parameters.

The proposed interactive disaggregation–aggregation procedure finds values of the model parameters that best restore the assignment examples provided by the decision maker. Finding values of all the model parameters at once, that is, the weights, all thresholds, category bounds and the cutting level λ used in ELECTRE TRI-B, requires, however, solving a hard nonlinear programming problem. To overcome this difficulty, one can decompose the inference procedure into a series of linear programmes specialized in finding values of subsets of these parameters. A computer implementation of this inference method with respect to weights and the $\lambda-$cutting level gave birth to a software tool called ELECTRE Tri ASSISTANT (Mousseau *et al*., 2000). The tool is also able to identify 'inconsistent judgements' in the assignment examples.

Let us notice that in all inference procedures concerning the ELECTRE TRI-B method, only the 'pessimistic' version of the assignment procedure was considered (the 'optimistic' version is even more difficult to model in terms of mathematical programming because it requires binary variables).

The *inference-based approaches* proposed after the work of Mousseau and Słowiński (1998) are the following:

1. Inferring the *weights* and the $\lambda-$cutting level of ELECTRE TRI-B by linear programming (the discriminating and the veto thresholds as well as the category bounds being fixed) (Mousseau *et al*., 2001). In this work, the authors consider the linear programming model of Mousseau *et al*. (2000), and perform several numerical experiments related to checking the behaviour of this inference disaggregation tool. These experiments show that $2n$ (n being the number of criteria) assignment examples are sufficient to infer adequately the weights and the $\lambda-$cutting level.

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2. Inferring the *bounds of categories* (Ngo The & Mousseau, 2002). This work deals with the possibility of inferring the bounds of categories of ELECTRE TRI-B. After making some simplifying assumptions, the authors developed linear programming and 0–1 linear programming models to infer the bounds.
3. Inferring *veto thresholds* (Dias & Mousseau, 2006). This work is a complement of the previous ones. The authors proposed mathematical programming models to assess veto thresholds for the original outranking relation and its two other variants, which may be used in ELECTRE methods, including ELECTRE III. In this case, the inference tools make use of linear programming, 0–1 linear programming, or separable programming.
4. Some *manageable disaggregation procedures* for valued outranking relations were proposed (Mousseau & Dias, 2006). The authors propose a modified definition of the valued outranking relation, preserving the original discordance concept. This modification *makes it easier* to solve inference problem via mathematical programming. These procedures can be used within ELECTRE III and ELECTRE TRI methods.
5. For some decision examples given by decision makers, there may be no feasible values of model parameters that would permit the model to represent these examples. We then say that the preference information is *inconsistent with respect to the model*. Resolving inconsistency is a problem of utmost importance, as shown in Mousseau *et al*. (2003, 2006). The authors propose algorithms for resolving inconsistency, where the decision makers must choose between different options of withdrawing or relaxing inconsistent examples. It should be noted, however, that unless inconsistency does not come from violation of dominance, it is not a fault of the decision maker but a deficiency of the preference model to restore the decision examples. Thus, instead of withdrawing or relaxing inconsistent examples, one should also consider the possibility of using a more adequate preference model (Figueira, 2009).

4.1.2. Inference-robustness based approaches. The *disaggregation–aggregation* approach for *inferring weights* and deriving *robust conclusions* in sorting problems was proposed in Dias *et al*. (2002). This work presents a new interactive approach that combines two different approaches, the inference-based approach with the robustness-based approach. It is also applied to ELECTRE TRI-B. The first approach was described in the previous subsection. The second approach considers a set of constraints with respect to the parameter values (weights and λ −cutting level), used to model the imperfect character of the information provided by the decision maker. Then, for each action, the best and worst categories compatible with the constraints are determined. This type of results allows to derive some robust conclusions about the assignments. The robustness analysis is used in this study to guide the decision maker through an interactive inference of weights and λ −cutting level.

4.1.3. Pseudo-robustness based approaches. Stability analysis or pseudo-robust conclusions based on Monte Carlo simulation methods, mainly for ranking and sorting problems (Tervonen *et al*., 2009a). The authors propose a new method SMAA-TRI on the basis of stochastic multiple criteria acceptability analysis (SMAA), for analysing the stability of some parameters of the ELECTRE TRI-B method. The method consists of analyzing finite spaces of uniformly distributed parameter values. Then, a Monte Carlo simulation is applied in these spaces for describing each action in terms of the share of parameter values that have been assigned to different categories. This is a kind of stability analysis that can be used to derive pseudo-robust conclusions. For each action, the result obtained is the share of parameter values for each category (in terms of percentage).

4.1.4. New concepts for robustness concerns. Although having a more general range of applicability, the contributions that will be described later should be able to bring answers to the robustness concerns, when applied to decision aiding using ELECTRE methods.

In Section 3.4 of Aissi and Roy (2010), the authors propose a measure of robustness, which is applied to ranking of potential actions $a \in A$ obtained when using ELECTRE III or ELECTRE IV, in the case where it is necessary to take into account a family $\hat{S}$ of scenarios (or of 'variable settings'). Let P_s denote a (partial or complete) order provided by ELECTRE with scenario $s \in \hat{S}$, and let $P = \left\{P_s \middle| s \in \hat{S}\right\}$. First, the authors consider the following measure of robustness:

$$r_\alpha(a) = \text{ Proportion of pre-orders } P_s \in \hat{S},$$

in which a occupies a position in the ranking at least equal to α; where α denotes an *a priori* fixed position. Under such basis, we can judge that action 'a is at least as robust as action a'', when $r_\alpha(a) \geqslant r_\alpha(a')$. Then, the authors proposed to improve this measure by taking into account another position in the ranking β

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(also defined *a priori*) to penalize the actions with a very bad position in certain scenarios. Thus, they propose the following robustness measure:

$$r_{\alpha\beta}(a) = r_\alpha(a) - \text{ Proportion } of\ P_s \in \hat{S},$$

in which action a occupies a position in the ranking greater than or equal to β.

The results obtained with this robustness measure (possibly supplemented by a sensitivity analysis with respect to the reference positions α and β) must lead to robust conclusions easily understandable by the decision maker (for more details on this subject see Section 6 in Chapter 1 in Zopounidis and Pardalos (2010)).

Still in Section 5.3 of Aissi and Roy (2010), the authors propose two frameworks intended to generalize an approach that was successfully used in two concrete cases by one of the authors. In these formal frameworks (using different ELECTRE methods), the approach allows to work out some conclusions and then recommendations answering to certain robustness concerns. The approach mainly aims at restricting the number of combinations of the options to be explored. This restriction is supported by putting in clear positions those combinations of options, which appear to have the most significant effect for answering robustness concerns.

In Chapter 1 of Zopounidis and Pardalos (2010), B. Roy introduces in Section 5 various suggestions and proposals for answering to certain robustness concerns by weakening the role of the worst case. These suggestions and proposals do not concern in particular the ELECTRE methods, but, at least for some of them, they can be useful.

4.2. Improvements and new approaches

This section presents the main novelties of ELECTRE-like methods, such as a concept of bi-polar outranking relations implemented in the RUBIS method (proposed by Birsdorff et al. (2008) as a generic identifier for bipolar-valued concordance-based decision aiding), the modelling of three different types of interaction among criteria, the research carried out to modify the credibility index through the use of the reinforced preference thresholds and the counter-veto thresholds, the ELECTRE TRI-C and ELECTRE TRI-NC methods, and the ELECTRE^{GKMS} method.

4.2.1. Bi-polar outranking based procedures. The concept of *bi-polar outranking relations* was proposed in Bisdorff *et al.* (2008) and implemented in the RUBIS software. The RUBIS method is a progressive MCDA method for choice problems. It is also an outranking based method. It is, however, based on a new concept of bi-polar outranking relation.

The bi-polar outranking index $\tilde{S} : A \times A \rightarrow [-1, 1]$ is defined as follows: for $(a,a') \in A \times A$,

$$\tilde{S}\left(a, a'\right) = \min\left\{\tilde{C}\left(a, a'\right), -V_1\left(a, a'\right), \ldots, -V_n\left(a, a'\right)\right\}$$

where

$$\tilde{C}\left(a, a'\right) = \sum_{\left\{j \in \mathcal{C}^S\left(a,a'\right)\right\}} w_j - \sum_{\left\{j \in \mathcal{C}^P\left(a',a\right)\right\}} w_j$$

and for all $g_j \in F$,

$$V_j\left(a, a'\right) = \begin{cases} 1 & \text{if } g_j(a) - g_j\left(a'\right) \leqslant -v_j\left(g_j(a)\right), \\ -1 & \text{if } g_j(a) - g_j\left(a'\right) > -wv_j\left(g_j(a)\right), \\ 0 & \text{otherwise} \end{cases}$$

where $wv_j(g_j(a))$ is a weak veto threshold such that $p_j(g_j(a)) < wv_j(g_j(a)) \leqslant v_j(g_j(a))$.

The maximum value +1 of the bi-polar outranking index is reached in the case of unanimous concordance, whereas the minimum value –1 is obtained either in the case of unanimous discordance, or if there exists a strong veto situation on at least one criterion. The median situation 0 represents a case of indetermination: either the arguments in favour of an outranking are compensated by those against it, or a positive concordance in favour of the outranking is outbalanced by a potential (weak) veto situation.

The semantics linked to this bi-polar outranking index is the following:

- $\tilde{S}\left(a, a'\right) = +1$ means that assertion 'aSa'' is *clearly validated*,
- $\tilde{S}\left(a, a'\right) > 0$ means that assertion 'aSa'' is *more validated than non-validated*,
- $\tilde{S}\left(a, a'\right) = 0$ means that assertion 'aSa'' is *undetermined*,
- $\tilde{S}\left(a, a'\right) < 0$ means that assertion 'aSa'' is *more nonvalidated than validated*,
- $\tilde{S}\left(a, a'\right) = -1$ means that assertion 'aSa'' is *clearly non-validated*.

On the basis of the bi-polar outranking index, a recommendation for choice problems is given by a procedure on the basis of five pragmatic principles ($\mathcal{P}_1$: the nonretainment for well-motivated reasons; $\mathcal{P}_2$: the minimal size, $\mathcal{P}_3$: the efficient and informative refinement, $\mathcal{P}_4$: the effective recommendation, and $\mathcal{P}_5$: the maximal credibility) and the theoretical concepts of hyperkernel and augmented chordless circuits in a digraph.

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4.2.2. Taking into account the interaction between criteria. The interaction between criteria is modelled through the *weights of the interaction coefficients* and the modifications in the concordance index (Figueira *et al*., 2009a). This work presents an extension of the comprehensive (overall) concordance index of ELECTRE methods, which takes the interaction among criteria into account. Three types of interactions have been considered: (*a*) mutual strengthening, (*b*) mutual weakening and (*c*) antagonism. The new concordance index correctly takes into account these three types of interactions by imposing such conditions as boundary, monotonicity and continuity. The following types of interactions were considered (let us notice that the cases *a*–*b* are mutually exclusive, but cases *a*–*c* and *b*–*c* are not). Let $\bar{C}\left(a^{'}Pa\right)$ denote the coalition of criteria that strongly opposes to the assertion 'a outranks a'':

a) *Mutual strengthening effect*
If both criteria g_i and g_j strongly, or even weakly, support the assertion aSa' (more precisely, $g_i, g_j \in \bar{C}\left(a^{'}Pa\right)$), we consider that their contribution to the concordance index must be greater than the sum of $k_i + k_j$, because these two weights represent the contribution of each of the two criteria to the concordance index when the other criterion does not support aSa'. We suppose that the effect of the combined presence of both g_i and g_j among the criteria supporting the assertion aSa' can be modelled by a mutual strengthening coefficient $k_{ij} > 0$, which intervenes algebraically in $c(a,b)$. Note that $k_{ij} = k_{ji}$.

b) *Mutual weakening effect*
If both criteria g_i and g_j strongly, or even weakly, support the assertion aSa' (more precisely, $g_i, g_j \in \bar{C}\left(a^{'}Pa\right)$), we consider that their contribution to the concordance index must be smaller than the sum of $k_i + k_j$, because these two weights represent the contribution of each of the two criteria to the concordance index when the other criterion does not support aSa'. We suppose that this effect can be modelled using a mutual weakening coefficient $k_{ij} < 0$, which intervenes algebraically in $c(a,a')$. Note that $k_{ij} = k_{ji}$.

c) *Antagonistic effect*
If criterion g_i strongly, or weakly, supports the assertion aSa', and criterion g_h strongly opposes to this assertion, we consider that the contribution of criterion g_i to the concordance index must be smaller than the weight k_i that was considered in the cases in which g_h does not belong to $C(a'Pa)$. We suppose that this effect can be modelled by introducing an antagonism coefficient $k^{'}_{ih} > 0$, which intervenes negatively in $c(a,a')$. Note that the presence of an antagonism coefficient $k^{'}_{ih} > 0$ is compatible with both the absence of antagonism in the reverse direction ($k^{'}_{hi} = 0$) and the presence of a reverse antagonism ($k^{'}_{hi} > 0$).
The antagonistic effect does not double the influence of the veto effect; in fact, they are quite different. If criterion g_h has a veto power, it will always be considered, regardless of whether g_i belongs to the concordant coalition or not. The same is not true for the antagonistic effect, which occurs only when criterion g_i belongs to the concordant coalition. Let us notice that a veto threshold expresses the power attributed to a given criterion g_j to be against the assertion 'a outranks a'', when the difference between performances $g_j(a')$ and $g_j(a)$ is greater than this threshold.

The authors demonstrated that the generalized index is able to take satisfactorily into account the three types of interactions or dependencies among criteria, and they also examined the links between the new concordance index and the Choquet integral. Nevertheless, this extension is appropriate only when the number of pairs of interacting criteria is rather small. Otherwise, we consider that the family of criteria should be rebuilt, because it contains too many interactions and (possibly) incoherencies.

4.2.3. The reinforced preference and the counter-veto effects. The credibility index $\sigma(a,a')$ of the outranking relation aSa' involves performance scales that are purely ordinal. For this reason, as soon as on criterion g_j, the difference of performances $g_j(a) - g_j(a')$ becomes greater than the preference threshold, the value of this difference does not influence the credibility of outranking of action a over action a'. If one would judge that a very large value of this difference obtains the meaning of 'very strong preference', then one could wish to take this judgement into account in the definition of the credibility of outranking of a over a'. To satisfy such a wish, Roy and Słowiński (2008) propose two complementary ways:

- The first one involves a new threshold called reinforced preference threshold: it corresponds to the value of the difference of performances $g_j(a) - g_j(a')$ that is 'judged meaningful' for considering criterion g_j as more important in the concordant coalition (by increasing its weight), comparing to the situation where (all things equal elsewhere) the difference of performances is smaller than this threshold (however, not smaller than the preference threshold);

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- The second one involves another threshold called counter-veto threshold (it is not necessarily equal to the previous one, as it has a different meaning, and it plays a different role): it corresponds to the value of the difference of performances $g_j(a) - g_j(a')$ that is 'judged meaningful' for weakening the mechanism of veto against the credibility of outranking (from the side of discordant criteria), comparing to the situation where (all things equal elsewhere) the difference of performances is smaller than this threshold (however, not smaller than the preference threshold).

After defining some principles and requirements for the new formula of the credibility index $\sigma(a,a')$ giving account of the aforementioned two ways, Roy and Słowiński provide the following proposal that satisfies these requirements.

Let $rp_j(g_j(a))$ denote the reinforced preference threshold for criterion g_j. When this threshold is crossed, the importance coefficient w_j in the formula for concordance index $c(a,a')$ should be replaced by $\omega_j w_j$, where $\omega_j > 1$ is called reinforcement factor. Let $\mathcal{C}^{RP}\left(a, a'\right)$ denote the set of criteria for which $g_j(a) > g_j(a') + rp_j(g_j(a))$. The new concordance index is then defined as follows,

$$\hat{c}\left(a, a'\right) = \frac{\sum_{\left\{j \in \mathcal{C}^{RP}\left(a,a'\right)\right\}} \omega_j w_j + \sum_{\left\{j \in \mathcal{C}^{S}\left(a,a'\right) \setminus \mathcal{C}^{RP}\left(a,a'\right)\right\}} w_j + \sum_{\left\{j \in \mathcal{C}^{Q}\left(a',a\right)\right\}} w_j \varphi_j}{\sum_{\left\{j \in \mathcal{C}^{RP}\left(a,a'\right)\right\}} \omega_j w_j + \sum_{\left\{j \in F \setminus \mathcal{C}^{RP}\left(a,a'\right)\right\}} w_j}$$

For any criterion g_j, $g_j \in F$, the two thresholds $rp_j(g_j(a))$ and $cv_j(g_j(a))$ can be chosen equal, and, moreover, one may wish to consider only one of the two effects; deleting an effect means giving to the corresponding threshold an infinite or very large value. Consequently, no order relation is imposed between these two thresholds.

The new formula for the index of the credibility of outranking $\hat{\sigma}\left(a, a'\right)$ can be substituted to similar formulae used in original versions of ELECTRE III, ELECTRE TRI-B, ELECTRE TRI-C and ELECTRE TRI-NC.

The assignment of values to the new thresholds $rp_j(g_j(a))$ and $cv_j(g_j(a))$ can be carried out in a constructive way of thinking about the model of decision problem at hand. One can use for this some protocols of inquiry similar to those proposed for assigning appropriate values to indifference and preference thresholds (Roy & Bouyssou, 1993), or to the weights (Figueira & Roy, 2002). These protocols involve few easy questions that do not require from the addressee to speculate about completely unrealistic situations. Another way could be to proceed via disaggregation–aggregation approach, so as to have thresholds $rp_j(g_j(a))$ and $cv_j(g_j(a))$ as compatible as possible with some exemplary pairwise comparisons of few real actions (Mousseau & Słowiński, 1998).

Let $cv_j(g_j(a))$ denote the counter-veto threshold for criterion g_j, and k the number of criteria for which this threshold has been crossed.

To give account of the reinforced preference and the counter-veto effects, the credibility index $\sigma(a,a')$ of the assertion aSa' has to be adequately adapted. For example, the credibility index $\sigma(a,a')$ defined in point 3.3.1, takes the following form:

$$\hat{\sigma}\left(a, a'\right) = c\left(a, a'\right) \left[\prod_{j \in \mathcal{J}\left(a,a'\right)} \frac{1 - d_j\left(a, a'\right)}{1 - c\left(a, a'\right)} \right]^{(1-k/n)},$$

where $j \in \mathcal{J}\left(a, a'\right)$ if and only if $d_j(a,a') \geqslant c(a,a')$. Again, $\hat{\sigma}\left(a, a'\right) \in [0, 1]$.

The way of introducing the two new effects is consistent with the handling of purely ordinal preference scales. Each of the two new thresholds is like a frontier representing a qualifier without any reference to a notion of quantity. The weights remain intrinsic weights and do not become substitution rates; the indifference and preference thresholds play exactly the same role as before.

The new formula could also be used outside ELECTRE methods, for example, to define similarity or closeness indices (Słowiński & Stefanowski, 1994; Słowiński & Vanderpooten, 1997; Słowiński & Vanderpooten, 2000), or to define filtering operators (Perny, 1998).

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4.2.4. The ELECTRE TRI-C and ELECTRE TRI-NC methods for sorting problems. ELECTRE TRI-C (Almeida-Dias *et al.*, 2010) is a new method for sorting problems designed for dealing with decision-aiding situations where each category from a completely ordered set is defined by a single characteristic reference action. The characteristic reference actions are co-constructed through an interactive process involving the analyst and the decision maker. ELECTRE TRI-C has been also conceived to verify a set of natural structural requirements (conformity, homogeneity, monotonicity and stability). The method makes use of two joint assignment rules, where the result is a range of categories for each action to be assigned.

Set A of the considered actions is either completely known *a priori* or may appear progressively during the decision-aiding process. The objective is to assign these actions to a set of completely ordered categories, denoted by $C_1, \ldots, C_h, \ldots, C_q$ with $q \geqslant 2$. The two joint rules, called descending rule and ascending rule, can be presented as follows:

Descending rule
Choose a credibility level $\lambda \in [0.5,1]$. Decrease h from $(q+1)$ until the first value t, such that $\sigma(a,b_t) \geqslant \lambda$:

- For $t=q$, select C_q as a possible category to assign action a.
- For $0<t<q$, if $\rho(a,b_t) > \rho(a,b_{t+1})$, then select C_t as a possible category to assign a; otherwise, select C_{t+1}. (ρ is a selection function).
- For $t=0$, select C_1 as a possible category to assign a.

Ascending rule
Choose a credibility level $\lambda \in [0.5,1]$. Increase h from 0 until the first value k, such that $\sigma(b_k,a) \geqslant \lambda$:

- For $k=1$, select C_1 as a possible category to assign action a.
- For $1<k<(q+1)$, if $\rho(a,b_k) > \rho(a,b_{k-1})$, then select C_k as a possible category to assign a; otherwise, select C_{k-1}.
- For $k=(q+1)$, select C_q as a possible category to assign a.

Each one of the two joint rules requires the selecting function $\rho(a,b_h)$, which allows to choose between the two consecutive categories where an action a can be assigned to. The results appear in one of the following forms, and the decision maker may choose:

1. A single category, when the two selected categories are the same;
2. One of the two selected categories, when such categories are consecutive;
3. One of the two selected categories or one of the intermediate categories, when such categories are not consecutive.

In Almeida-Dias *et al.* (2012), ELECTRE TRI-C method is generalized to ELECTRE TRI-NC method where each category is defined by a set of several reference characteristic actions, rather than one. This aspect is enriching the definition of each category and allows to obtain more narrow ranges of categories to which an action can be assigned to, than the ELECTRE TRI-C method. The joint assignments rules are similar to the previous ones.

4.2.5. The robust ordinal regression approach: ELECTRE^{GKMS} method. The inference-based approaches to ELECTRE methods presented in Subsection 3.1 have been recently extended to handle in a special way the robustness concerns. More precisely, in Greco *et al.* (2009, 2011), the authors consider the inference-based approach to ELECTRE methods using the *Robust Ordinal Regression* (ROR) (Greco *et al.*, 2010). In ROR, the preference parameters of a decision model are inferred from holistic preference comparisons of some reference actions made by the decision maker. In consequence, one obtains in general many sets of values of preference model parameters representing this preference information; however, in previous inference-based approaches, only one specific set was selected and used to work out a recommendation. Because the selection of one among many sets of parameter values compatible with the preference information provided by the decision maker is rather arbitrary, the ROR approach proposes taking into account all these sets to give a recommendation in terms of *necessary* and *possible* consequences of applying all the compatible preference models on the considered set of actions (Greco *et al.*, 2008; Figueira *et al.*, 2009b). With respect to ELECTRE methods, the ROR approach is applied in the method ELECTRE^{GKMS}, where the possible and the necessary outranking relations are calculated as follows. Given an ordered pair of actions $(a,a') \in A \times A$, *a necessarily* outranks a', which is denoted by $aS^N a'$, if for all compatible sets of parameter values, a outranks a', whereas *a possibly* outranks a', denoted by $aS^P a'$, if for at least one compatible set of parameter values, a outranks a'. After exploiting the necessary outranking relation in the similar way as in the original ELECTRE methods, one obtains a robust recommendation, because it is supported by all outranking models compatible with the holistic preference information. The ELECTRE^{GKMS} method has been adapted also to the case of group decision making, and called

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ELECTRE^{GKMS}-GROUP method (Greco *et al*., 2009; Greco *et al*., 2011). In this case, several decision makers cooperate in a decision problem to make a collective decision. Decision makers share the same 'description' of the decision problem (the same set of actions, evaluation criteria and performance matrix). Each decision maker provides his or her own preference information, composed of pairwise comparisons of some reference actions. The collective preference model accounts for preferences expressed by each decision maker. Let us denote the set of decision makers by $\mathcal{D} = \{d_1, , d_2, \ldots, , d_p\}$. For each decision maker $d_r \in D' \subseteq D$, we consider all compatible outranking models. Four situations are interesting for an ordered pair $(a, a') \in A \times A$:

1. $aS^{N,N}(D')a'$: $aS^N a'$ for all $d_r \in D'$,
2. $aS^{N,P}(D')a'$: $aS^N a'$ for at least one $d_r \in D'$,
3. $aS^{P,N}(D')a'$: $aS^P a'$ for all $d_r \in D'$,
4. $aS^{P,P}(D')a'$: $aS^P a'$ for at least one $d_r \in D'$.

4.3. Axiomatic and meaningfulness analysis

This section is devoted to theoretical foundations of ELECTRE methods, concerning their axiomatization and the meaningfulness of statements they provide with respect to different performance scales of the considered criteria.

4.3.1. Axiomatic analysis. Concerns about axiomatic basis of ELECTRE methods have been described in a long series of papers started in the last millennium (Bouyssou, 1986; Bouyssou & Vansnick, 1986; Pirlot, 1997; Bouyssou *et al*., 1997). The contributions on this topic were continued in this millennium (Dubois *et al*., 2003; Bouyssou & Pirlot, 2002a). We will not review in detail all the works on the axiomatic analysis of ELECTRE methods, but we will concentrate our attention on contributions related to conjoint measurement analysis of ELECTRE methods carried out by Bouyssou and Pirlot on one hand, and Greco, Matarazzo and Słowiński, on the other hand.

Greco *et al*. (2001a) introduced the first conjoint measurement model of an ELECTRE method, namely, ELECTRE I. Let $X = X_1 \times X_2 \times \ldots \times X_n$ be a product space, where X_j is the value set of criterion $j = 1, 2, \ldots, n$. Let (x_j, z_{-j}), $x_j \in X_j$ and $z_{-j} \in X_{-j} = \prod_{i=1, i \neq j}^{n} X_i$, denote an element of X equal to z except for its j^{th} coordinate being equal to x_j. Analogously, let $(x_{\bar{A}}, z_{-\bar{A}})$, $x_{\bar{A}} \in X_{\bar{A}} = \prod_{j \in \bar{A}}$ and $z_{-\bar{A}} \in X_{-\bar{A}} = \prod_{j \notin \bar{A}} X_j$, $\bar{A} \subseteq \{1, 2, \ldots, n\}$, denote an element of X equal to $x_{\bar{A}}$ for coordinates $j \in \bar{A}$ and to $z_{-\bar{A}}$ for coordinates $j \notin \bar{A}$. A comprehensive outranking relation $\succsim$ is defined on X such that $x \succsim y$ means that 'x is at least as good as y'. The symmetric part of $\succsim$ is the indifference relation denoted by $\sim$, whereas the asymmetric part of $\succsim$ is the preference relation denoted by $\succ$. The only minimal requirement imposed on $\succsim$ is its reflexivity. In the following, for each $j = 1, \ldots, n$, we consider a partial outranking relation $\succsim_j$, such that $x_j \succsim_j y_j$ means 'criterion j is in favour of the comprehensive outranking of x over y'.

For each ordered pair $(x,y) \in X$, let $S(x,y) = \{j \mid x_j \succsim_j y_j\}$.

We say that a comprehensive outranking relation $\succsim$ on X and the partial outranking relations $\succsim_j$, $j = 1, \ldots, n$, constitute a concordance structure if and only if for all $x, y, w, z \in X$:

$$[S(x,y) \supseteq S(w,z)] \Rightarrow [w \succsim z \Rightarrow x \succsim y].$$

Greco *et al*. (2001a) presented the following result.

Theorem 1 (Greco *et al*., 2001a)
The three following propositions are equivalent:

1) *for each* $x_j, y_j, u_j, v_j, w_j, z_j \in X_j, a_{-j}, b_{-j}, c_{-j}, d_{-j} \in X_{-i}, j = 1 \ldots, n$,

$$\text{(A)}\ \left[(x_j, a_{-j}) \succsim (y_j, b_{-j}) \text{ and } (u_j, c_{-j}) \succsim (v_j, d_{-j})\right] \Rightarrow \left[(x_j, c_{-j}) \succsim (y_j, d_{-j}) \text{ or } (w_j, a_{-j}) \succsim (z_j, b_{-j})\right],$$

and

$$\text{(B)}\ (x_j, a_{-j}) \succsim (y_j, b_{-j}) \Rightarrow (x_j, a_{-j}) \succsim (x_j, b_{-j});$$

2) *there exists a partial outranking relation* $\succsim_j$ *for each criterion* $j = 1, \ldots, n$, *such that the comprehensive outranking relation* $\succsim$ *on* X *is a concordance structure*;
3) *there exists*

   – *a partial outranking relation* $\succsim_j$ *for each criterion* $j = 1, \ldots, n$,
   – *a set function (capacity)* $v: 2^{\{1,\ldots,n\}} \rightarrow [0,1]$, *such that* $v(\emptyset) = 0$, $v(\{1, \ldots, n\}) = 1$ *and for each* $\bar{A} \subseteq \bar{B} \subseteq \{1, \ldots, n\}$, $v(\bar{A}) \leqslant v(\bar{B})$, *and*
   – *a threshold* $t \in]0, 1[$ *such that* $v(S(x,y)) \geqslant t \Leftrightarrow x \succsim y$.

ELECTRE methods are based not only on the concordance relation but also on the discordance relation. For each criterion $j = 1, \ldots, n$, there is defined a veto relation V_j, such that for each $x_j, y_j \in X_j$, $x_j V_j y_j$ means that 'the preference of y_j over x_j is so strong that, for all $a_{-j}, b_{-j} \in X_{-j}$, it is not true that $(x_j, a_{-j}) \succsim (y_j, b_{-j})$', that is, (x_j, a_{-j}) cannot be as good as (y_j, b_{-j}).

We say that a comprehensive outranking relation $\succsim$ on X is a concordance structure with veto if and only if for all $x, y, w, z \in X$:

$$\left[S(x,y)\supseteq S(w,z) \text{ and non}\left(x_jV_jy_j\right) \text{ for all } j=1,\ldots,n\right]\Rightarrow\left[w\succsim z\Rightarrow x\succsim y\right].$$

Greco *et al*. (2001a) presented also the following result.

Theorem 2 (Greco *et al*., 2001a)
The three following propositions are equivalent:

1) *for each* $x_j, y_j, u_j, v_j, w_j, z_j \in X_j, a_{-j}, b_{-j}, c_{-j}, d_{-j}, e_{-j}, f_{-j} \in X_{-j}, j=1,\ldots,n$,

$$(\mathrm{A}')\left[\left(x_j,a_{-j}\right)\succsim\left(y_j,b_{-j}\right) \text{ and } \left(u_j,c_{-j}\right)\succsim\left(v_j,d_{-j}\right) \text{ and } \left(w_j,e_{-j}\right)\succsim\left(z_j,f_{-j}\right)\right]\Rightarrow\left[\left(x_j,c_{-j}\right)\succsim\left(y_j,d_{-j}\right) \text{ or } \left(w_j,a_{-j}\right)\succsim\left(z_j,b_{-j}\right)\right],$$

and aforementioned condition (B) holds;
2) *there exists a partial outranking relation* $\succsim_j$ *and a veto relation* V_j *for each criterion* $j=1,\ldots,n$, *such that the comprehensive outranking relation* $\succsim$ *on X is a concordance structure with the veto relation*;
3) *there exists*,

- *a partial outranking relation* $\succsim_j$ *for each criterion* $j=1,\ldots,n$,
- *a set function (capacity)* $v:2^{\{1,\ldots,n\}}\rightarrow[0,1]$, *such that* $v(\varnothing)=0$, $v(\{1,\ldots,n\})=1$ *and for each* $\bar{A}\subseteq\bar{B}\subseteq\{1,\ldots,n\}$, $v(\bar{A})\leqslant v(\bar{B})$ *and*
- *a threshold* $t\in]0,1[$ *such that*,

$$v(S(x,y))\geqslant t \text{ and } V(x,y)=\varnothing\Leftrightarrow x\succsim y.$$

Bouyssou and Pirlot (2002) *introduced another axiomatic analysis of ELECTRE I that includes a certain number of results aiming at presenting the ELECTRE I method as a special case of their non-additive nontransitive model*.

Theorem 3 (Bouyssou & Pirlot, 2005)
The aforementioned Theorem 1 holds $\succsim_{-j}$ *being complete for* $j=1\ldots,n$ *also when Proposition 1 of Theorem 1 is replaced by the following one*:
(1′) for each $x_j, y_j, w_j, z_j\in X_j, a_{-j}, b_{-j}, c_{-j}, d_{-j}\in X_{-j}$, $j=1\ldots,n$,

$$(\mathrm{RC2})\quad\left[\left(x_j,a_{-j}\right)\succsim\left(y_j,b_{-j}\right) \text{ and } \left(y_j,c_{-j}\right)\succsim\left(x_j,d_{-j}\right)\right]\Rightarrow\left[\left(z_j,a_{-j}\right)\succsim\left(w_j,b_{-j}\right) \text{ or } \left(w_j,c_{-j}\right)\succsim\left(z_j,d_{-j}\right)\right],$$

$$(\mathrm{UC})\quad\left[\left(x_j,a_{-j}\right)\succsim\left(y_j,b_{-}\right) \text{ and } \left(z_j,c_{-j}\right)\succsim\left(w_j,d_{-j}\right)\right]\Rightarrow\left[\left(y_j,a_{-j}\right)\succsim\left(x_j,b_{-j}\right) \text{ or } \left(x_j,c_{-j}\right)\succsim\left(y_j,d_{-j}\right)\right],$$

$$(\mathrm{LC})\quad\left[\left(x_j,a_{-j}\right)\succsim\left(y_j,b_{-j}\right) \text{ and } \left(y_j,c_{-j}\right)\succsim\left(x_j,d_{-j}\right)\right]\Rightarrow\left[\left(y_j,a_{-j}\right)\succsim\left(x_j,b_{-j}\right) \text{ or } \left(z_j,c_{-j}\right)\succsim\left(w_j,d_{-j}\right)\right].$$

The axioms of the first result, however, interact with the axioms of their nonadditive and nontransitive model (Bouyssou & Pirlot, 2002), *and, therefore, they produced another result*.

Theorem 4 (Bouyssou & Pirlot, 2007)
The aforementioned Theorem 1 holds also when Proposition 1′ of Theorem 3 is replaced by the following one:
(1″) for each $x_j, y_j, w_j, z_j\in X_j, a_{-j}, b_{-j}, c_{-j}, d_{-j}\in X_{-j}, j=1,\ldots,n$,

$$(\mathrm{RC1})\quad\left[\left(x_j,a_{-j}\right)\succsim\left(y_j,b_{-j}\right) \text{ and } \left(z_j,c_{-j}\right)\succsim\left(w_j,d_{-j}\right)\right]\Rightarrow\left[\left(x_j,c_{-j}\right)\succsim\left(y_j,d_{-j}\right) \text{ or } \left(z_j,a_{-j}\right)\succsim\left(w_j,b_{-j}\right)\right],$$

$$(\mathrm{M1})\quad\left[\left(x_j,a_{-j}\right)\succsim\left(y_j,b_{-j}\right) \text{ and } \left(z_j,c_{-j}\right)\succsim\left(w_j,d_{-j}\right)\right]\Rightarrow\left[\left(y_j,a_{-j}\right)\succsim\left(x_j,b_{-j}\right) \text{ or } \left(w_j,a_{-j}\right)\succsim\left(z_j,b_{-j}\right) \text{ or } \left(x_j,c_{-j}\right)\succsim\left(y_j,d_{-j}\right)\right],$$

$$(\mathrm{M2})\quad\left[\left(x_j,a_{-j}\right)\succsim\left(y_j,b_{-j}\right)\text{and } \left(y_j,c_{-j}\right)\succsim\left(x_j,d_{-j}\right)\right]\Rightarrow\left[\left(y_j,a_{-j}\right)\succsim\left(x_j,b_{-\mathrm{j}}\right) \text{ or } \left(z_j,a_{-j}\right)\succsim\left(w_j,b_{-j}\right) \text{ or } \left(z_j,c_{-j}\right)\succsim\left(w_j,d_{-j}\right)\right],$$

*and aforementioned condition (**RC2**) holds*.
Finally, Bouyssou and Pirlot (2009) *consider also the veto condition, proposing the following result*.

Theorem 5 (Bouyssou & Pirlot, 2009)
The aforementioned Theorem 2 holds also when Proposition 1 of Theorem 2 is replaced by the following one:
for each $x_j, y_j, w_j, z_j\in X_j, a_{-j}, b_{-j}, c_{-j}, d_{-j}, e_{-j}, f_{-j}\in X_{-j}, j=1,\ldots,n$,

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(M3) $[(x_j,a_{-j})\succsim(y_j,b_{-j})$ and $(y_j,c_{-j})\succsim(x_j,d_{-j})$ *and* $(z_j,e_{-j})\succsim(w_j,f_{-j})]$
$$\Rightarrow$$
$[(y_j,a_{-j})\succsim(x_j,b_{-j})$ or $(z_j,a_{-j})\succsim(w_j,b_{-j})$ *or* $(z_j,c_{-j})\succsim(w_j,d_{-j}))$

and aforementioned conditions (**RC1**), (**RC2**) and (**M1**) hold.

The approach of Bouyssou and Pirlot (2009) has the merit of putting the axiomatic basis of ELECTRE methods in the larger context of their general nonadditive and nontransitive model. However, their conditions are more numerous and complex than the conditions proposed by Greco, Matarazzo and Slowinski (2001a).

4.3.2. Representing preferences by decision rules. In Greco *et al*. (2002), an equivalence of preference representation by conjoint measure and decision rules using the Dominance-based Rough Set Approach (DRSA) (Greco *et al*., 2001) was demonstrated for choice and ranking problems. One of the most important conclusions in this context is that ELECTRE I method can be represented in terms of DRSA. In this case, for all $a\in A$ and for all $g_j\in F$, $q_j(g_j(a))=p_j(g_j(a))$, such that $Q_j=\varnothing$, and $d_j(a,a')\in\{0,1\}$. Then, the set of decision rules describing the aggregation procedure of ELECTRE I has the following form:

$$\text{if } aS_{j_1}a' \text{ and } \ldots aS_{j_p}a' \text{ and } \ldots aV^c_{j_p+1}a' \text{ and } \ldots aV^c_{j_n}a',\ \text{then } aSa'$$

where $aV^c_j a'$ means that $d_j(a,a')=0$ (i.e. there is no veto with respect to criterion $g_j\in F$) and

$$w_{j_1}+\ldots+w_{j_p}\geqslant s$$

with s being a specific concordance threshold. Not all the aforementioned decision rules are necessary to obtain a representation of the outranking relation S on A, because it is enough to consider only those decision rules that involve subsets $\tilde{F}=\{g_{j_1},\ldots,,g_{j_p}\}\subseteq F$ including no $g_i\in\tilde{F}$ for which

$$w_{j_1}+\ldots+w_{j_p}-w_i\geqslant s.$$

With the use of this result, Greco *et al*. (2002) proposed a methodology to infer preference model parameters (weights and veto thresholds) of ELECTRE methods from a set of decision rules obtained by DRSA.

4.3.3. A conjoint measurement analysis of a simplified version of ELECTRE TRI-B. An axiomatic analysis of a simplified variant of ELECTRE TRI-B has been proposed in Bouyssou and Marchant (2007, 2007), in the framework of conjoint measurement theory. This variant only takes into account the 'pessimistic' assignment rule and does not make use of veto thresholds; preference and indifference thresholds are considered equal.

From a technical point of view, the authors make use of conjoint measurement techniques to work with partitions, instead of binary relations. This aspect of dealing with the problem was first proposed by Goldstein (1991) and after generalized by Greco *et al*. (2001). Based, moreover, on the concepts of conjoint measurement theory, these authors analyse a certain type of 'non-compensatory sorting methods' close to the 'pessimistic' version of ELECTRE TRI-B, and make a comparison with other sorting methods. They proved that the simplified version of ELECTRE TRI-B is noncompensatory. This result does not hold, however, for the 'optimistic' version of ELECTRE TRI-B with the same simplifications.

Some hints to elicit parameters from assignment examples within the framework of the studied version of ELECTRE TRI-B were also provided in their work.

To give an axiomatic basis to ELECTRE TRI-B, they considered the following simplified model. Consider a twofold partition $\langle\mathcal{A},\mathcal{U}\rangle$ of X, which means that the two sets $\mathcal{A}$ and $\mathcal{U}$ are nonempty and disjoint, and that their union makes the entire set X. For the sake of simplicity, one can imagine $\mathcal{A}$ as a set of all good actions, and $\mathcal{U}$ as a set of all bad actions. In ELECTRE TRI-B, the sorting of action $x\in X$ is based on comparison of x with profile p separating the categories, using outranking relation S. Then, in the 'pessimistic' version of ELECTRE TRI-B, for all $x\in X$,

$$x\in\mathcal{A}\Leftrightarrow xSp,$$

whereas in the 'optimistic' version of ELECTRE TRI-B,

$$x\in\mathcal{A}\Leftrightarrow not(pPx),$$

where P is the asymmetric part of S, that is, xSp and *not* (pSx). A partition $\langle\mathcal{A},\mathcal{U}\rangle$ has a representation in the noncompensatory sorting model if:

- for all $j=1,\ldots,n$, there is a set $\mathcal{A}_j\subseteq X_j$,
- there is a subset $\mathcal{F}$ of 2^N, such that, for all $I,J\in 2^N$, $N=\{1,\ldots,n\}$,

$$[I\in\mathcal{F} \text{ and } I\subseteq J]\Rightarrow J\in\mathcal{F},$$

such that, for all $x\in X$,

$$x\in\mathcal{A}\Leftrightarrow\{j\in N|x_j\in\mathcal{A}_j\}\in\mathcal{F}.$$

Bouyssou and Marchant (2007) presented the following result.

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Theorem 6 (Bouyssou & Marchant, 2007)
A partition $\langle \mathcal{A}, \mathcal{U} \rangle$ *has a representation in the noncompensatory sorting model if and only iffor each* x_j, $y_j \in X_j$ *and all* $a_{-j}, b_{-j} \in X_{-j}, j = 1 \ldots, n,$

$$\text{(Linear)}\ \left[\left(x_j, a_{-j}\right) \in \mathcal{A} \text{ and } \left(y_j, b_{-j}\right) \in \mathcal{A}\right] \Rightarrow \left[\left(y_j, a_{-j}\right) \in \mathcal{A} \text{ or } \left(x_j, b_{-j}\right) \in \mathcal{A}\right],$$

$$(2 - graded)\ \left[\left(x_j, a_{-j}\right) \in \mathcal{A} \text{ and } \left(y_j, a_{-j}\right) \in \mathcal{A} \text{ and } \left(y_j, b_{-j}\right) \in \mathcal{A}\right] \Rightarrow \left[\left(x_j, b_{-j}\right) \in \mathcal{A} \text{ and } \left(z_j, a_{-j}\right) \in \mathcal{A}\right].$$

It is interesting to note that the same axioms have been given by Słowiński et al. (2002) *as an axiomatic basis to the sorting procedure on the basis of the Sugeno integral* (1974). *Therefore, the noncompensatory sorting model is equivalent to the sorting model on the basis of the Sugeno integral.*

Bouyssou and Marchant (2007) *considered also a noncompensatory sorting model with veto, that augments the aforementioned noncompensatory sorting model by consideration of sets* $\mathcal{V}_j \subseteq X_j, j = 1, \ldots, n$, *such that for all* $x \in X$,

$$x \in \mathcal{A} \Leftrightarrow \left[\left\{j \in N | x_j \in \mathcal{V}_j\right\} \in \mathcal{F} \text{ and } \left\{j \in N | x_j \in \mathcal{V}_j\right\} = \varnothing\right].$$

Indeed, Bouyssou and Marchant (2007) *presented the following result.*

Theorem 7 (Bouyssou & Marchant, 2007)
A partition $\langle \mathcal{A}, \mathcal{U} \rangle$ *has a representation in the noncompensatory sorting model with veto if only if for each* x_j, $y_j \in X_j$ and all $a_{-j}, b_{-j} \in X_{-j}, j = 1, \ldots, n,$

$$(3v - graded)\left[\left(x_j, a_{-j}\right) \in \mathcal{A} \text{ and } \left(y_j, a_{-j}\right) \in \mathcal{A} \text{ and } \left(y_j, b_{-j}\right) \in \mathcal{A} \text{ and } \left(z_j, c_{-j}\right) \in \mathcal{A}\right) \Rightarrow \left[\left(x_j, b_{-j}\right) \in \mathcal{A} \text{ and } \left(z_j, a_{-j}\right) \in \mathcal{A}\right],$$

*and aforementioned condition (**Linear**) holds.*

In Bouyssou and Marchant (2007), *this approach has been extended to give an axiomatic basis to the noncompensatory sorting in the case of more than two classes.*

4.3.4. The meaningfulness of ELECTRE methods. In Martel and Roy (2006), the authors analyse the meaningfulness of the assertions of the type 'a outranks a' for such and such method', in particular, for ELECTRE methods.

The notion of meaningfulness (Suppes, 1959) comes from the measurement theory. This theory (Luce *et al*., 1990) deals with the way one can represent certain information (in particular, information of qualitative nature) coming from a given category of phenomena through a set of numerical values, in such a way that this representation must adequately reflect certain properties of the considered category of phenomena.

To obtain a meaningful assertion (with respect to a considered category of phenomena) based on the computations that make use of the numerical representation, it is necessary that its validity or nonvalidity will not be affected when one uses another adequate measure or way of representing the phenomena. Indeed, meaningfulness in MCDA is related to invariance of results with respect to some admissible transformation of the performance scales.

In ELECTRE methods, when there are no ambiguity zones (all the preference thresholds are equal to the indifference thresholds), the meaningfulness is ensured, even for purely ordinal scales. If, for some criteria, the indifference thresholds are strictly lower than the preference thresholds, the loss of meaningfulness is locally restricted to the ambiguity zones between these thresholds. Consequently, ELECTRE methods are meaningful without requirement for criteria to possess interval scales (Martel & Roy, 2006).

4.4. Other relevant issues

This section is devoted to other issues related to ELECTRE methods, that do not fit the previous sections, but, nevertheless are important for several reasons.

4.4.1. The relative importance of criteria. The metaphor of weight is very often a source of misunderstanding (Roy, 2010). Knowing the weight of different objects allows to line them up from the heaviest to the lightest one. Similarly, the talk about the (relative) weight of two criteria assumes implicitly that the assertion 'this criterion is more important than the other one' makes a sense. It leads to suppose that the weight of a criterion has an intrinsic character, that is to say that it depends only on the point of view reflected by it, and does not depend on the manner in which it is modelled (the nature of the scales, the range of the scales, the possible unit, . . .). Very often, researchers and practitioners had the opportunity to notice that it is in such a way that a decision maker uses (even before talking to him or her) the expression 'weight of a criterion'. This parameter holds different names, according to the type of model in which it intervenes. It is, nevertheless, the term *weight* that is the most often used.

It is, in general, the notion of more or less big importance between two criteria that makes naturally sense in the head of the decision makers. Simos (1990) proposed a procedure that was further revised by Figueira and Roy (2002). These authors proposed a method, called Simos-Roy-Figueira (SRF), for assessing the importance coefficients of criteria having exactly the aforementioned meaning. They also stressed the fact that SRF must not be used for the coefficients (called inappropriately weights) of a weighted-sum, and that it must be reserved for intrinsic weights (independent on the very nature of the scales) corresponding to the number of voices that could be allocated to every criterion in a voting process. It should be noted that SRF first exploits the ordinal character of the criteria scales, which means that the units and the range of the scales play no role in the assessment of the importance coefficients (to be more rigorous, a very local and minimal role). As mentioned before, the decision makers, who express themselves spontaneously about the notion of importance of criteria, make, in general, no link between this notion and the nature of the scales. The MCAP used to aggregate this information must reflect such a fact adequately.

4.4.2. Concordant outranking with criteria of ordinal significance. In Bisdorff (2004), a new contribution to robustness concerns in MCDA was proposed. More precisely, a complete preorder π on the family of criteria F is considered, which is a ranking of significance of criteria, to be taken into account in the construction of the comprehensive outranking relation S. The weights are π-compatible if for all $g_j, g_{j'} \in F$, $w_j = w_{j'}$ if g_j and $g_{j'}$ have the same rank of significance in π, and $w_j > w_{j'}$ if g_j has a higher rank of significance than $g_{j'}$ in π. If for $(a,a') \in A \times A$ the concordance index $c(a,a') > 0.5$, for every π-compatible set of weights, there is an ordinal concordance of a over a', which is denoted by $aC_\pi a'$.

4.4.3. Evolutionary approaches. Evolutionary algorithms are starting to be used to deal with large scale problems, as well as, to mitigate the complexity of some computations in ELECTRE methods, mainly because of some nonlinearities existing in the formulas used in these methods.

In Doumpos *et al.* (2009), an evolutionary approach was proposed to deal with construction of outranking relations in the context of ELECTRE TRI-B.

In Leyva-López *et al.* (2008), a new MCDA method was proposed for ranking problems. It makes use of the ELECTRE III method to build a fuzzy outranking relation and exploit it through the application of a multi-objective genetic algorithm.

4.4.4. The EPISSURE method for the assessment of nonfinancial performance. A new approach making use of the ELECTRE TRI-B method is presented in Chapter 13 of (Zopounidis & Pardalos, 2010). This new approach, called EPISSURE (splice in French, which is a nautical term meaning a joint made by splicing) has been designed by André (2009) for evaluating nonfinancial performances of companies.

Because of the fierce competition in markets among companies and institutions, and because of a strong pressure by international entities to take into account other kinds of performance criteria than financial ones, there was a need of a new approach to the evaluation of nonfinancial performance of the companies. EPISSURE responds to this need.

Two normative principles were laid down *ex-ante* to ground the approach:

- *Principle 1*: The approach must be hierarchical, that is, classified into successive levels, wherein the levels match a hierarchy of responsibilities *vis-à-vis* the successive aggregates of performance that contribute to the performance summary.
- *Principle 2*: At each hierarchical level (except perhaps for some at the lowest levels), the evaluations rely on ordinal verbal scales. The number of degrees on the scales must be adjusted to its matching levels; the number of degrees must be high enough to mirror evolutions and be understandable by the stakeholders operating at the said level.

A consultation process, called a framed consultation process, is an integral part of the EPISSURE approach. As any other consultation approach, the objective is that the different stakeholders involved in the evaluation reach a common outlook.

The EPISSURE approach was tested and set up within several companies for the purpose of evaluating sponsorship projects and deciding on their follow-up. The results seem to indicate that this approach is decidedly appropriate for evaluating nonfinancial performance. Another application concerning evaluation of the environmental performance of the Company *Total* is described in André and Roy (2007).

4.4.5. Group decision aiding. In Damart *et al.* (2007), the authors propose a framework for group decision aiding, when groups are willing to cooperate. It is based

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on an inference-based approach (see Subsection 3.1) to the ELECTRE TRI-B method. The implemented procedure is of an interactive nature, and it is based on a 'rule' that preserves the coherence of judgements about the sorting examples at both the individual and the group level. As mentioned in point 3.2.5, another inference-based approach to group decision with ELECTRE methods, has been proposed as ELECTRE^{GKMS}-GROUP method (Greco *et al.*, 2009; Greco *et al.*, 2011). It employs robust ordinal regression to work with all outranking models compatible with holistic preference information.

4.4.6. Recent applications. In what follows, we present some recent applications of ELECTRE methods.

1. Sorting cropping systems (Arondel & Girardin, 2000).
2. Land use suitability assessment (Joerin *et al.*, 2001).
3. Greenhouse gases emission reduction (Georgopoulou *et al.*, 2003).
4. Risk zoning of an area subjected to mining-induced hazards (Merad *et al.*, 2004).
5. Participatory decision making on the localization of waste-treatment plants (Norese, 2006).
6. Material selection of bipolar plates for polymer electrolyte membrane fuel cell (Shanian & Savadogo, 2006).
7. Assisted reproductive technology (Matias, 2008).
8. Promotion of social and economic development (Autran-Gomes *et al.*, 2009).
9. Sustainable demolition waste management strategy (Roussat *et al.*, 2009).
10. Assessing the risk of nanomaterials (Tervonen *et al.*, 2009b).

5. CONCLUDING REMARKS

ELECTRE methods have a long history of successful real-world applications with considerable impact on human decisions. Several application areas can be pointed out (see Figueira *et al.* (2005)): agriculture and forest management, energy, environment and water management, finance, military, project selection (call for tenders), transportation, medicine, nanotechnologies, . . . As every MCDA method, ELECTRE methods have also some theoretical limitations. This is why, when applying these methods, analysts should first check if their theoretical characteristics respond to the characteristics of the context in which they will be used.

In this paper, we tried to show that research on ELECTRE methods is not a dead field. Rather the opposite, it is still evolving and gains acceptance thanks to new application areas, new methodological and theoretical developments, as well as user-friendly software implementations.

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